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LETTERS TO THE EDITORS

THERMAL CONDUCTIVITY OF TWO-PHASE HETEROGENEOUS SUBSTANCES

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MANY theoretical expressions have been derived for evaluating the effective thermal conductivity of heterogeneous substances; a historical review is given by Godbee and Ziegler [1]. As mentioned in their paper, the expressions fall into two categories viz. Exact Solutions and Simplified Solutions. The latter are obtained by reducing the problem of calculating the effective thermal conductivity from that of solving a partial differential equation to that of solving an ordinary differential equation. The exact solutions, originated in Maxwell's and Lord Rayleigh's work, usually yield good agreement with experimental results on a two-phase system of relatively low concentration of discontinuous phase. However, they have not yielded good agreement in the high concentration range. Hence, Bruggeman [2] modified the Maxwell–Rayleigh equation and extended its use to the high concentration range. His equation is

$$1 - V_d = \frac{k_d - k_e}{k_d - k_c}\left(\frac{k_c}{k_e}\right)^{\frac{1}{3}} \qquad (1)$$

where k_e is the effective thermal conductivity, k_d and k_c are the thermal conductivities of the discontinuous and continuous phases respectively, and V_d is the fractional volume of the discontinuous phase. Most of the other expressions fall into the category of the simplified solutions, and they have not been compared with equation (1).

Cheng and Vachon [3] recently developed Tsao's model [4] and predicted the thermal conductivity of dispersed systems within 8 per cent of the available experimental data. I wish to point out that equation (1) can also predict it with similar accuracy. Moreover, Bruggeman's equation has no restriction for use and agrees well even with expressions for powder systems as shown in Fig. 1.

To sum up, Bruggeman's equation is recommended for evaluating the effective thermal conductivity of two phase heterogeneous substances without any restrictions.

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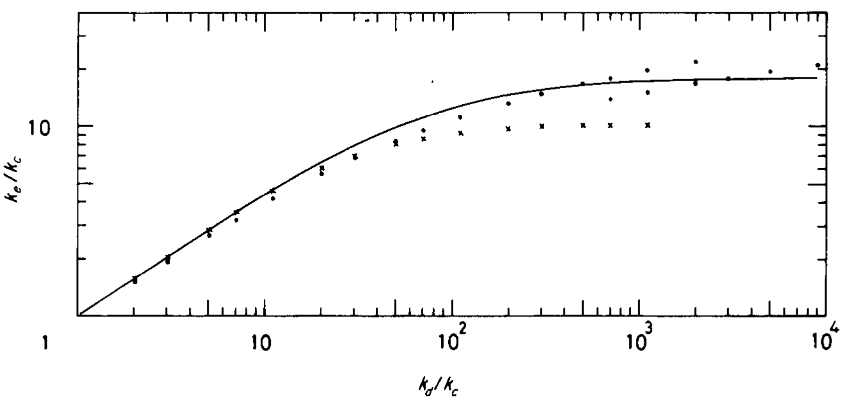

FIG. 1. The effective thermal conductivity for the case of $V_d = 0{\cdot}62$. The full curve is Bruggeman's equation; ● relation of [5] and [6], with which the paraboloidal revolution model [7] agrees in the range of $k_d/k_c \lesssim 100$; × Fricke's relation [8] with de Vries' value [9] which underestimates k_e when k_d/k_c is very large (see [10]).

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PIONEER PAPER BY W. NUSSELT

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It was a very great pleasure to see again in print in the July issue, the pioneer paper by W. Nusselt covering the basic similarity relationships for forced and natural convection. Some of your English speaking readers may be interested to know that an authorized translation of the paper has been available for many years and that it has been used at universities in teaching the principles of similarity and dimensional analysis: the paper remains one of the very best for teaching purposes. Reprinting in the International Journal of Heat and Mass Transfer is particularly pleasing to the translators since it supports their earlier views of the classic value of the paper.

The translation (No. 681, $2.50 per copy) is available from the Publication Section, Division of Building Research, National Research Council, Ottawa, Canada.

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